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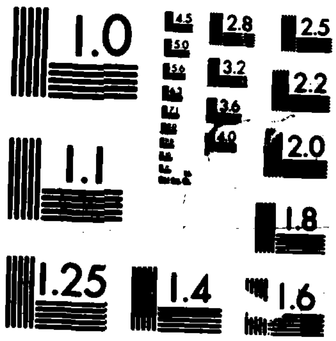

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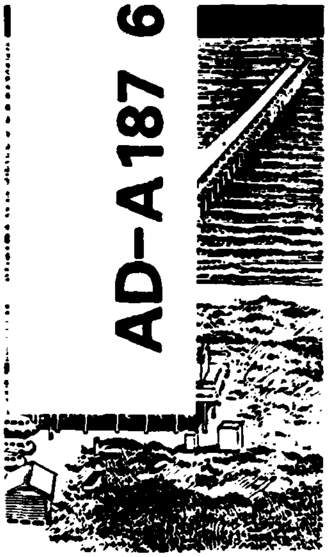

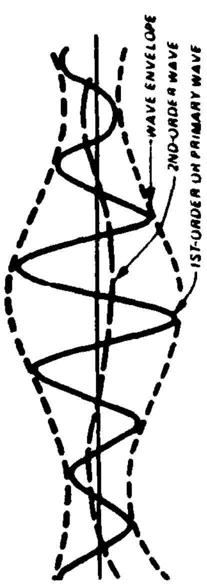

TECHNICAL REPORT CERC-87-14

WAVE GROUP ANALYSIS BASED ON KIMURA'S METHOD

by

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Coastal Engineering Research Center

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September 1987

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The computer program KIMUR5 was developed to predict wave group run lengths based on Kimura's method. This method, which is based on an assumption of mutual correlation between successive waves, has been demonstrated to be superior to Goda's method. Wave grouping is especially significant in shallow-water laboratory models of harbor resonance, beach runup, and slow-drift motions of moored structures. Descriptions, theoretical development, documentation, and a listing of the computer program are provided. Copies of the computer program KIMUR5 and associated subroutines are available upon request.

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PREFACE

This report is a by-product of the Laboratory Simulation of Spectral and Directional Spectral Waves Work Unit 31762, Coastal Flooding and Storm Protection Program, Civil Works Research and Development, at the US Army Engineer Waterways Experiment Station's (WES's) Coastal Engineering Research Center (CERC). The Office, Chief of Engineers, US Army Corps of Engineers (OCE), provided funds for the research herein. Messrs. John H. Lockhart, Jr., and John G. Housley, OCE, were Technical Monitors for the Coastal Flooding and Storm Protection Program. Dr. Charles L. Vincent is CERC Program Manager.

This research was originally performed during the author's visit to the Hydraulics Laboratory of the National Research Council of Canada (NRCC), Ottawa, Ontario, Canada, and he extends special thanks to Dr. Etienne Mansard and Mr. Dan Pelletier of NRCC for their assistance.

Mr. Michael J. Briggs, Research Hydraulic Engineer, Wave Processes Branch (CW-P), Wave Dynamics Division (CW), CERC, prepared this report with assistance from Ms. Mary L. Hampton, Civil Engineering Technician, CW-P, CERC, under direct supervision of Mr. Douglas G. Outlaw, Chief, CW-P; and under general supervision of Mr. C. Eugene Chatham, Chief, CW; Mr. Charles C. Calhoun, Jr., Assistant Chief, CERC; and Dr. James R. Houston, Chief, CERC. Ms. Shirley J. Hanshaw, Information Products Division, Information Technology Laboratory, edited this report.

COL Dwayne G. Lee, CE, was Commander and Director of WES during report publication. Dr. Robert W. Whalin was Technical Director.

CONTENTS

WAVE GROUP ANALYSIS BASED ON KIMURA'S METHOD

PART I: INTRODUCTION

1. High sea waves tend to appear in groups rather than individually. Engineers are finding that this grouping has important ramifications on the motions and resonances of moored structures and vessels, harbor resonance, stability and overtopping of shore protection structures, and surf beat. Because of the nature of wave grouping, its prediction, control, and analysis are especially important in shallow-water laboratory basins such as the Coastal Engineering Research Center's (CERC's) directional spectral wave basin.

2. This report is the result of research conducted at the Hydraulics Laboratory of the National Research Council of Canada (NRCC), Ottawa, Ontario, Canada, from 4-20 September 1985. During this time, the original version of computer program KIMUR5 was researched, written, debugged, and tested. The program calculates wave group run probabilities, lengths, means, and standard deviations using Kimura's method (Kimura 1980). His method, which is based on the assumption that successive wave heights are mutually correlated, has been demonstrated (van Vledder 1983b; Thomas, Baba, and Harish 1986) to be superior to Goda's method.

3. This report describes wave grouping and the differences in theory between Goda's and Kimura's methods. Additionally, it documents the computer program KIMUR5 and serves as a user's manual for program organization, input/output operations, and test cases. Finally, it provides recommendations for future expansion of the program. A copy of the computer program KIMUR5 and associated subroutines is available upon request.

PART II: WAVE GROUPING

4. Bounded long waves are associated with the occurrence of wave groups and produce a variation of the mean water level which produces a setdown under wave groups and a setup between groups (Figure 1). Longuet-Higgins and Stewart (1962) first described this second-order or nonlinear effect which results from a variation in the radiation stress (proportional to the square of the local wave height). The forced long wave propagates at the group velocity of the primary waves. Its amplitude is proportional to the square of the wave envelope and is relatively small, but it can increase dramatically as the depth and frequency decrease and wave groupiness increases. The second-order wave system propagates with phase opposite to the envelope of the first-order system. A crest of the second-order system coincides with a trough of the wave group envelope. Second-order currents are also created by the occurrence of wave groups. These currents are important in the calculation of resistance forces of structures and mooring forces for vessels.

5. A succession of high waves that exceeds some arbitrary threshold value (typically median, mean, or significant wave height) is called a run of high waves, and the number of waves in this run is the run length (Figure 2). The total or complete run is the combination of the run of high waves followed by the run of low waves (i.e. succession of waves which fall below the threshold value). The total run is analogous to the zero-upcrossing period of the wave profile, except that the time series is composed of individual wave heights rather than surface elevations. Reference to a wave group assumes that a run of high waves is intended.

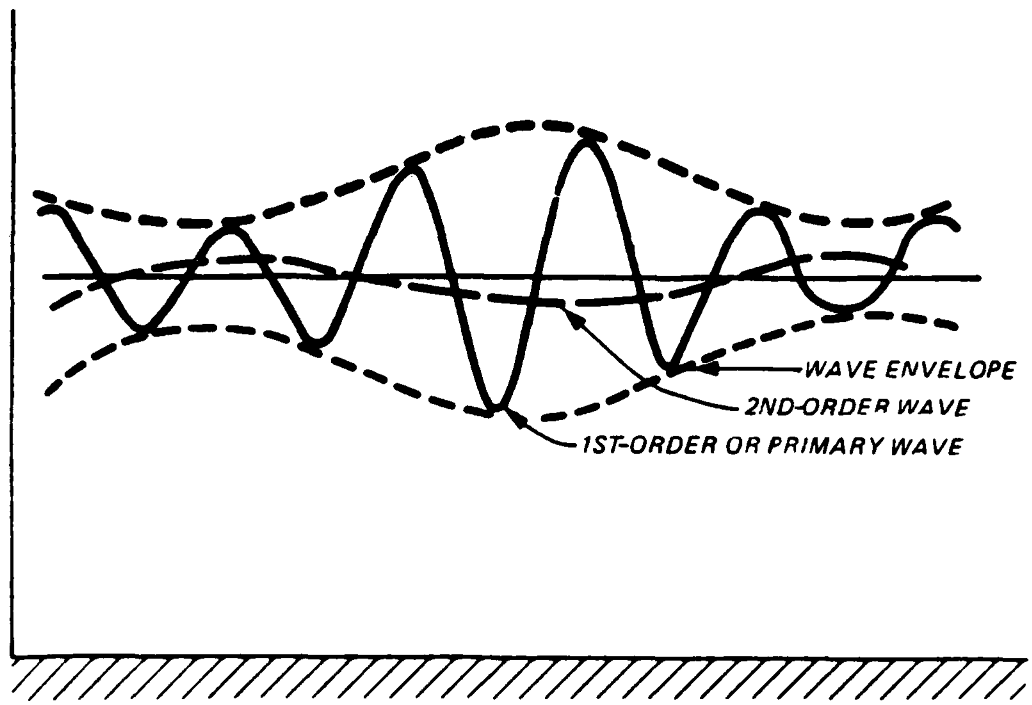

Figure 1. Schematic of wave grouping and second-order effects

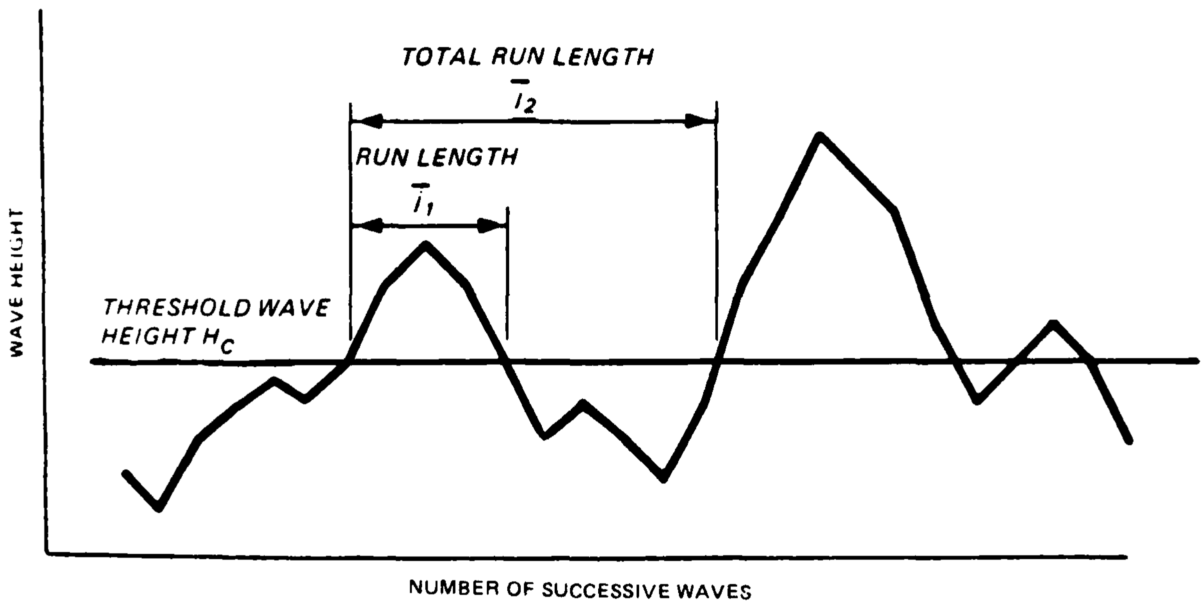

Figure 2. Definition of wave group run lengths

PART III: THEORY OF WAVE GROUPS

Goda's Method

6. Goda's method assumes that successive wave heights are uncorrelated or independent. The derivation is based on probability theory. If the probability p* that a wave height H will be greater than a threshold value H_c is

$$p = \text{Prob}\ [H > H_c] \tag{1}$$

then the probability that H will be less than or equal to H_c is given by

$$q = \text{Prob}\ [H \le H_c] = 1 - p \tag{2}$$

since $p + q = 1$. (Some authors use the symbol H_* for the cutoff or threshold wave height.)

Run of high wave statistics

7. The probability distribution of a run of j_1 successive high waves follows the geometric distribution

$$P(j_1) = p^{j_1-1} q \qquad j_1 = 1,2,3... \tag{3}$$

which means that $j_1 - 1$ successive waves will exceed the threshold, and the j_1^{th} wave will not. The probability of successive wave heights exceeding the threshold is equal to the product of separate probabilities for each event because the wave heights are independent in Goda's (1985) theory.

8. According to Goda (1985), the mean run length $\overline{j_1}$ and standard deviation of the length of a run of high waves $\sigma(j_1)$ are, respectively,

---

* For convenience, symbols and abbreviations are listed in the Notation (Appendix D).

$$\overline{j_1} = E[j_1] = \sum_{j_1=1}^{\infty} j_1 P(j_1) = \frac{1}{q} \tag{4}$$

$$\sigma(j_1) = E\left[j_1^2\right] - E^2[j_1] = \frac{\sqrt{p}}{q} \tag{5}$$

where E[ ] is the expectation operator.

Total run statistics

9. For a total run of j_2 successive waves, the probability distribution $P(j_2)$, mean $\overline{j_2}$, and standard deviation $\sigma(j_2)$ are, respectively,

$$P(j_2) = \frac{pq}{p - q} p^{j_2-1} - q^{j_2-1} \qquad j_2 = 2,3,4,... \tag{6}$$

$$\overline{j_2} = E[j_2] = \frac{1}{p} + \frac{1}{q} = \frac{1}{pq} \tag{7}$$

$$\sigma(j_2) = E[j_2^2] - E^2[j_2] = \sqrt{\frac{p}{q^2} + \frac{q}{p^2}} \tag{8}$$

Example calculations

10. Example calculations based on Equations 1 to 8 for the probabilities and mean and standard deviations for the run of high waves and total run are listed in Table 1 for various values of threshold wave height. According to Goda's model, measured average group length larger than the values given in Table 1 indicates a higher level or degree of wave group formation.

Kimura's Method

11. Kimura's model assumes that successive wave heights are mutually correlated and form a Markov Chain. The concept of mutual correlation implies that successive waves are dependent or correlated. A high wave rarely appears by itself; rather, it is more likely to be followed by other high waves. It

Table 1

Theoretical Wave Group Statistics for Various Wave Height Thresholds

Goda's Model

Quantity	Threshold Wave Height: Median	Mean	Significant	Highest 1/10
p	0.500	0.456	0.135	0.039
q	0.500	0.544	0.865	0.961
$\overline{j_1}$	2.00	1.84	1.16	1.04
$\sigma(j_1)$	1.41	1.24	0.42	0.21
$\overline{j_2}$	4.00	4.03	8.58	26.55
$\sigma(j_2)$	2.00	2.04	6.92	25.01

seems that the waves have a "memory" which dictates that one high wave will be followed by another high wave rather than a low wave.

12. Transition probabilities for simultaneous exceedance and non-exceedance of the threshold wave height are calculated based on the ratio of one- and two-dimensional (i.e. joint or bivariate) Rayleigh probability density functions (PDF). From these Rayleigh-derived transition probabilities, the probability of a run of various lengths, the average run length, and the standard deviation of the run length are calculated for a run of successive high waves and a total run.

Markov Chain

13. A fundamental assumption of Kimura's model is that successive wave heights form the Markov Chain. The transition equation describing the Markov Chain is

$$P_n = P_0 p^n \tag{9}$$

where

P_n = distribution after n-time transitions

P_0 = initial distribution

p = transition probability matrix

If a threshold wave height H_c is selected (i.e. mean, median, significant, or highest 1/10 wave height), waves with height H will fall into one of two states or groups as shown below

State	Condition
1	$H \le H_c$
2	$H > H_c$

The initial distribution is then

$$P_0 = (0,1) \tag{10}$$

since a run of high waves begins when State 2 is first reached. The transition probability matrix is given by

$$p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{11}$$

where the individual elements or conditional probabilities are defined as

$$\begin{aligned} p_{11} &= \text{Prob}\left[H_{i+1} \le H_c \mid H_i \le H_c\right] \\ p_{12} &= \text{Prob}\left[H_{i+1} > H_c \mid H_i \le H_c\right] \\ p_{21} &= \text{Prob}\left[H_{i+1} \le H_c \mid H_i > H_c\right] \\ p_{22} &= \text{Prob}\left[H_{i+1} > H_c \mid H_i > H_c\right] \end{aligned} \tag{12}$$

and H_i and H_{i+1} represent successive wave heights. Thus, p_{11} is the probability that neither successive wave exceeds the threshold H_c, and p_{22} is the probability of simultaneous exceedance by both wave heights. By substituting Equations 10 and 11 into Equation 9, processing for n-time transitions and precluding the transition probabilities from State 1, we obtain the probability distribution for the run of high waves. If simple induction continues, the probability distribution of the total run can be determined in a similar fashion.

Transition probabilities

14. The other fundamental assumption of Kimura's method is that the transitional or conditional probabilities for wave height, p_{11} and p_{22}, are defined in terms of the Rayleigh distribution. The conditional probabilities are

$$p_{11} = \frac{\int_0^{H_c} \int_0^{H_c} p(H_1,H_2)dH_1dH_2}{\int_0^{H_c} q(H_1)dH_1} \tag{13}$$

$$p_{22} = \frac{\int_{H_c}^{\infty} \int_{H_c}^{\infty} p(H_1,H_2)dH_1dH_2}{\int_{H_c}^{\infty} q(H_1)dH_1} \tag{14}$$

where

$p(H_1,H_2)$ = joint or two-dimensional Rayleigh PDF for two successive wave heights

H_1,H_2 = dummy wave height variables

$q(H_1)$ = Rayleigh PDF for individual wave heights

15. The PDF $q(H_1)$ defined in terms of the root-mean-square wave height H_r and the mean wave height H_m is

$$q(H_1) = \frac{2H_1}{H_r^2} \exp\left(- \frac{H_1^2}{H_r^2}\right) = \frac{\pi}{2} \frac{H_1}{H_m^2} \exp\left(- \frac{\pi}{4} \frac{H_1^2}{H_m^2}\right) \tag{15}$$

16. The Rayleigh joint PDF is similarly defined by Kimura (1980) based on earlier work of Rice (1944, 1945) and Uhlenbeck (1943) as

$$p(H_1, H_2) = \frac{4H_1H_2}{(1 - \kappa^2)H_r^4} \exp\left[- \frac{1}{(1 - \kappa^2)} \frac{H_1^2 + H_2^2}{H_r^2} \right] I_o \left[\frac{2\kappa}{(1 - \kappa^2)} \frac{H_1H_2}{H_r^2} \right]$$

$$= \frac{\pi^2 H_1H_2}{4(1 - \kappa^2)H_m^4} \exp\left[- \frac{\pi}{4(1 - \kappa^2)} \frac{H_1^2 + H_2^2}{H_m^2} \right] I_o \left[\frac{\pi\kappa}{2(1 - \kappa^2)} \frac{H_1H_2}{H_m^2} \right] \quad (16)$$

where κ is the correlation parameter and $I_o[\ ]$ is a modified Bessel function of zeroth order.

Correlation parameter

17. To solve for the joint Rayleigh PDF and the associated transition probabilities p_{11} and p_{22}, the correlation parameter κ is required. Kimura (1980) and some authors define it in terms of the variable ρ as

$$\kappa = 2\rho \quad (17)$$

Uhlenbeck (1943) showed that the correlation parameter κ is related to the correlation coefficient $R_{hh}(1)$, a measure of the degree of correlation or dependence between successive wave heights, by

$$R_{hh}(1) = \frac{E(\kappa) - \frac{(1 - \kappa^2)}{2} K(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}} \quad (18)$$

where $E(\ )$ and $K(\ )$ are complete elliptic integrals of the first and second kind, respectively. Some authors use γ_h to define the correlation coefficient. The correlation coefficient range is 0 to 1.0. A value of zero corresponds to the Goda model. Several investigators (Goda 1985) have calculated correlation coefficients of 0.24 for successive wind waves and 0.5 - 0.8 for swell. The amount of correlation tends to increase with higher wave heights and narrower wave spectra.

18. Battjes (1974) demonstrated that an infinite series representation for the elliptic integrals could be used to approximate Equation 18 as

$$R_{hh}(1) = \frac{\pi}{4(4 - \pi)} \left(\kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^6}{64} + \ldots \right) \quad (19)$$

If this relation is inverted, the correlation parameter κ can be determined given the correlation coefficient $R_{hh}(1)$ as follows:

$$\kappa^2 = R - \frac{R^2}{16} - \frac{R^3}{128} - \ldots \quad (20)$$

where

$$R = \left[\frac{4(4 - \pi)}{\pi} \right] R_{hh}(1) \quad (21)$$

Battjes (1974) found that this approximation is very good for correlation coefficients less than 0.7 to 0.8. From 0.8 to 1.0 the difference, although slight, is still noticeable.

Methods for determining correlation parameter

19. The correlation parameter can be determined in one of four ways:

a. Time Domain Method 1: assumed correlation coefficient.

b. Time Domain Method 2: autocorrelation technique.

c. Frequency Domain Method 1: Goda's spectral peakedness parameter.

d. Frequency Domain Method 2: Battjes' spectral derivation.

20. Assumed correlation coefficient. The assumed correlation coefficient method is the one currently coded in the computer program KIMUR5. The input value of the correlation coefficient $R_{hh}(1)$ is based on field measurements for similar wave conditions as the wave height time series to be analyzed. The correlation parameter κ is determined indirectly using Equations 19 and 20 above.

21. Autocorrelation technique. In the autocorrelation technique the correlation coefficient $R_{hh}(k)$ is first calculated from a zero-meaned, measured, or simulated wave height time series. The correlation parameter κ is determined indirectly using Equations 19 and 20 as before. The autocorrelation function estimate is normalized by the variance of the wave height time

series to give the correlation coefficient defined as

$$R_{hh}(k) = \frac{1}{\sigma_H^2} \frac{1}{N - k} \sum_{i=1}^{N-k} H_i H_{i+k} \qquad k = 1,2,3,... \tag{21}$$

where N is the total number of points in the wave height time series, and σ_H is the standard deviation of the series. The lag k is the difference in number between wave heights and is equal to 1 for successive wave heights. For every other wave height, the correlation coefficient would be written as $R_{hh}(2)$, every third wave height $R_{hh}(3)$, etc. The dependency between wave heights has been found by several investigators (van Vledder 1983a) to decrease rapidly as the lag is increased beyond successive wave heights (i.e. $R_{hh}(1)$).

22. Goda's spectral peakedness model. The spectral peakedness model is a frequency domain model based on a relationship between wave grouping and spectral form investigated by Goda (1970, 1976), Yamaguchi (1981), and Kimura (1980) among others. It is based on the spectral peakedness factor Q_p defined by Goda as

$$Q_p = \left(\frac{2}{m_o^2}\right) \int_0^\infty f\, S^2(f)\, df \tag{22}$$

where

m_o = zeroth moment of the time series

f = frequency

$S(f)$ = spectral estimate of the surface elevation

Goda (1985) found the spectral peakedness parameter to be insensitive to the high frequency cutoff used in spectral analysis. Its value ranges between 1 for white noise, 2 for wind waves, and 4 to 8 for swell conditions. The investigators mentioned above found that the average group length increases as the peakedness parameter increases, and a narrow spectrum has a greater degree of grouping than a widebanded spectrum.

23. Based on field wave data and numerical simulations for large values

of Q_p , Ewing (1973) proposed an approximately linear relationship between the mean run length $\overline{j_1}$ and the spectral peakedness parameter for a given cutoff or threshold wave height H_c as follows:

$$\overline{j_1} = \frac{Q_p}{H_c} \sqrt{2m_o} \tag{23}$$

24. Goda (1970) proposed the relationship between the correlation coefficient $R_{hh}(1)$ and the wave peakedness parameter Q_p as shown in Figure 3 (obtained from numerical simulations).

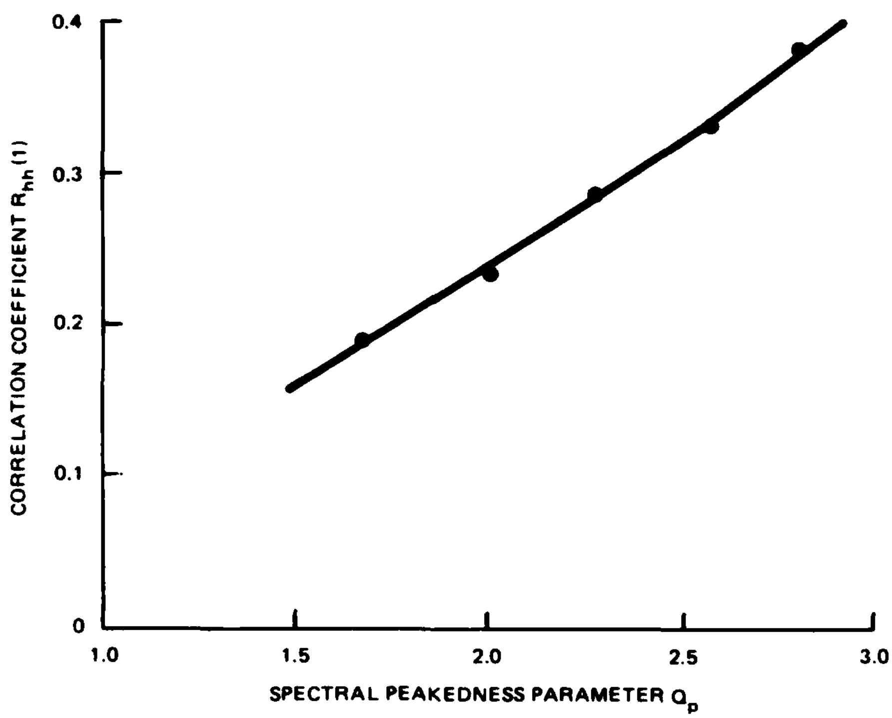

Figure 3. Relation between correlation coefficient and spectral peakedness parameter

25. <u>Battjes' spectral derivation.</u> Based on earlier work of Arhan and Ezraty (1978) on correlations with joint PDF's and Rice (1944, 1945) on theoretical envelope statistics, Battjes and van Vledder (1984) showed that the correlation parameter κ can be calculated spectrally by

$$\kappa = \frac{1}{m_o}\left\{\left[\int_o^\infty S(f)\cos(2\pi f\, T_m)\, df\right]^2 + \left[\int_o^\infty S(f)\sin(2\pi f\, T_m)\, df\right]^2\right\}^{0.5} \tag{24}$$

where T_m is the mean wave period obtained from zero-crossing or spectral analysis. In this sense, the correlation parameter κ is a measure of the spectral width, and Battjes and van Vledder (1984) noted that it is more "robust" than Goda's spectral peakedness parameter Q_p since it is not biased by sampling variability.

Run of high wave statistics

26. In Kimura's model, the probability of a run of successive high waves of run length j_1 is defined in terms of the transition probability p_{22} as

$$P(j_1) = p_{22}^{\,j_1-1}\,(1 - p_{22}) \qquad j_1 = 1,2,3,\ldots. \tag{25}$$

27. The mean $\overline{j_1}$ and standard deviation $\sigma(j_1)$ are, respectively,

$$\overline{j_1} = \frac{1}{(1 - p_{22})} \tag{26}$$

and

$$\sigma(j_1) = \frac{\sqrt{p_{22}}}{(1 - p_{22})} \tag{27}$$

Similarity exists among Equations 25 to 27 and Equations 3 to 5 for Goda's method in which p_{22} and $(1 - p_{22})$ replace p and q, respectively.

Total run statistics

28. For a total run, the probability distribution $P(j_2)$, mean $\overline{j_2}$, and standard deviation $\sigma(j_2)$ are, respectively,

$$P(j_2) = \frac{(1 - p_{11})(1 - p_{22})}{p_{11} - p_{22}}\left(p_{11}^{\,j_2-1} - p_{22}^{\,j_2-1}\right) \qquad j_2 = 2,3,4,\ldots \tag{28}$$

$$\overline{j_2} = \frac{1}{(1 - p_{11})} + \frac{1}{(1 - p_{22})} \tag{29}$$

$$\sigma(j_2) = \left[\frac{p_{22}}{(1 - p_{22})^2} + \frac{p_{11}}{(1 - p_{11})^2} \right]^{1/2} \tag{30}$$

Again, there is similarity with the total run statistics defined for Goda's model in Equations 6 to 8.

Comparison of Methods

29. Goda's model for wave group run lengths assumes that wave heights are independent or uncorrelated, although Rayleigh distributed. Rye (1974), Kimura (1980), and others have shown that wave heights are positively correlated. Thus, in comparisons with actual field measurements for varying wave environments (including wind wave generation in storms) by several investigators, Goda's method yields a constant value for several values of the correlation coefficient that seriously underpredict the degree of wave groupings. These comparisons of field measurements with Goda's model values for median H_{med} and significant H_s wave height threshold values are listed in Table 2 (van Vledder 1983a) which shows that the measured values for the run lengths are greater than those Goda predicted.

30. Table 3 shows the results of some computer simulations by Kimura (1980) for the group lengths of a run of high waves for spectra of various peakedness and uniform phase distributions. Goda's and Kimura's theoretical values for five different correlation coefficients are compared with the simulated data for threshold wave heights equal to the mean and significant wave height. Kimura's model shows a strong agreement with the data, while Goda's model gives a constant value that underpredicts the group length.

31. Table 4 contains analogous results by Goda (1983) for the group lengths of a run of high waves using measured data representative of long traveled swell with a narrow spectrum and high correlation coefficients. Again, there is serious underprediction of the Goda model and the reasonable correspondence between actual and predicted run lengths using the Kimura model.

Table 2

Comparison of Measured Average Group Lengths With Goda's Model

Run of High Waves

Investigator	Location	Time Period	Threshold Wave Height H_{med}	Threshold Wave Height H_s
	Theoretical Data			
Goda's model (1970)			2.00	1.16
	Measured Data			
Wilson and Baird (1972)	Nova Scotia	May-Jul	--	1.49
Rye (1974)	Norway	Oct-Dec	--	1.35
Goda (1976)	Japan	--	2.54	1.42
Dattatri, Raman, and Jothishankar 1977	India	Aug	2.23	1.34

Table 3

Comparison of Goda's and Kimura's Models with Simulated Data

Average Group Lengths for Run of High Waves

$R_{hh}(1)$	Threshold Wave Height: Mean, Goda	Mean, Kimura	Mean, Simulated	Significant, Goda	Significant, Kimura	Significant, Simulated
0.19	1.84	2.08	2.20	1.15	1.33	1.28
0.23	1.84	2.15	2.29	1.15	1.37	1.29
0.29	1.84	2.28	2.34	1.15	1.44	1.29
0.33	1.84	2.37	2.42	1.15	1.50	1.37
0.38	1.84	2.46	2.45	1.15	1.57	1.53

Table 4

Comparison of Goda's and Kimura's Models with Measured Data

Average Group Lengths for Run of High Waves

	Threshold Wave Height					
	Mean			Significant		
$R_{hh}(1)$	Goda	Kimura	Simulated	Goda	Kimura	Simulated
0.630	1.84	3.50	3.77	1.15	2.08	2.02
0.688	1.84	3.84	4.15	1.15	2.29	2.49
0.694	1.84	3.89	4.42	1.15	2.31	2.21

PART IV: COMPUTER PROGRAM DESCRIPTION

32. This section presents the documentation for the main program KIMUR5 and the eleven associated subroutines required. The intent is to provide the user with the documentation necessary to run the program. Appendix A contains a listing of the program and subroutines. Appendix B contains a listing of the symbols used in the computer program. Appendix C lists definitions of parameters for each subroutine and gives other documentation including descriptions, calling statement, calls to and by the subroutine, and references.

Program Specifications

33. Table 5 summarizes the program specifications for program KIMUR5.

Table 5

Program Specifications for Program KIMUR5

Specification	Description
Computer	DEC VAX 11/750
Location	USAE Waterways Experiment Station, CERC
Operating system	VAX/VMS version 4.2
Language	Fortran 77
Structure	Interactive, modular, top down
Documentation	Self-documenting
Subroutines	Eleven, shelf-contained (Subroutines BESI & QSF from NRCC Scientific Library)
Input	Interactive with prompts, logical unit 5
Output	Disk File KIMUR.OUT, logical unit 2
Accuracy	Single precision (Subroutine BESI requires double precision)
Operating procedure	Compile: FORTRAN KIMUR5
	Link: LINK KIMUR5
	Run: RUN KIMUR5
	Input: Enter 5 Input values at keyboard
	Output: TYPE or PRINT KIMUR.OUT

Solution Procedure

34. The computer code is presently in the form of a main driver program and eleven associated subroutines. Figure 4 is a flowchart illustrating the basic steps involved in program calculations. Table 6 lists the hierarchy of the individual subroutines in the program. A brief description of each subroutine is contained in Table 7. The steps indicated in Figure 4 (1-D = one dimensional; 2-D = two dimensional) are described in the paragraphs below.

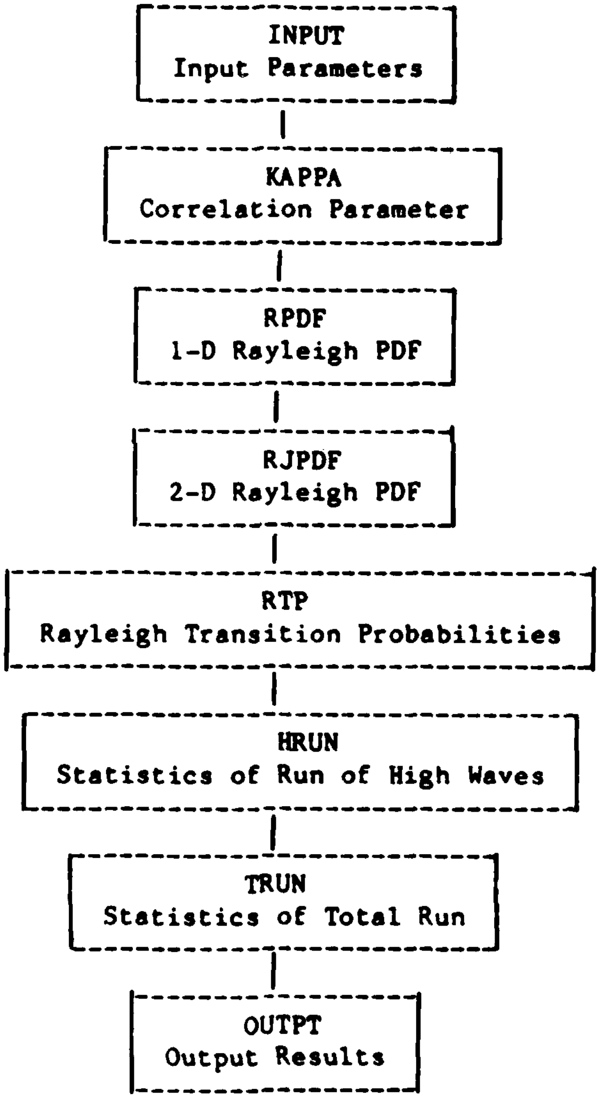

Figure 4. Flowchart of Program KIMUR5

Table 6

Hierachy of Program KIMUR5

Main Program	Subroutines Level 1	Level 2	Level 3
KIMUR5	INPUT		
	KAPPA		
	RPDF		
	RJPDF	BESI	
	RTP	RTPI	QSF
	HRUN		
	TRUN		
	OUTPT		

Table 7

Description of Subroutines in Program KIMUR5

Name	Description
INPUT	Queries user for input parameters
KAPPA	Calculates correlation parameter K given correlation coefficient RHH(1) using series approximation method of Battjes
RPDF	Calculates 1-D Rayleigh PDF Q(H1) for individual wave heights
RJPDF	Calculates 2-D Rayleigh joint PDF P(H1,H2)
BESI	Calculates Bessel function I_o of zeroth order (NRCC Scientific Subroutine Library)
RTP	Calculates Rayleigh transition probabilities P11 and P22
RTPI	Integrates 1-D and 2-D Rayleigh PDF's Q(H1) and P(H1,H2)
QSF	Computes vector integral values for a given equidistant table of function values using combination of Simpson's and Newton's 3/8 Rules (NRCC Scientific Subroutine Library)
HRUN	Calculates run of high wave group statistics of probability of different run lengths, mean run length, and standard deviation of run length
TRUN	Calculates total run group statistics of probability of various run lengths, mean run length, and standard deviation of run length
OUTPT	Outputs results to disk file for display and archival

Correlation parameter

35. The first step in the solution procedure is the calculation of the correlation parameter K from the input correlation coefficient RHH1. Subroutine KAPPA performs this operation using the infinite series approximation for the elliptic integrals given in Equation 20.

Rayleigh probability density function

36. The second step is the calculation of the 1-D Rayleigh PDF by Subroutine RPDF. The Rayleigh PDF given in Equation 15 for the mean wave height is programmed with the dummy wave height variable H1 defined as

$$H1 = N * DELH \qquad N = 0,1,2,\ldots NH \tag{31}$$

Descriptions of the symbols used in the computer program are contained in Appendix B.

Joint Rayleigh probability density function

37. The next step in the solution procedure is the calculation of the joint Rayleigh PDF by Subroutine RJPDF using the mean wave height form of Equation 16. For ease of programming, it is calculated in terms of three factors as

$$P(H1,H2) = A * B * C \tag{32}$$

where the A factor is a constant term, the B factor is an exponential term, and the C factor is the modified Bessel function of zeroth order. Again, dummy wave height interval variables H1 and H2 are used and defined in the range

$$\begin{aligned} H1 &= N * DELH \\ & \qquad\qquad N = 0,1,2,\ldots NH \\ H2 &= N * DELH \end{aligned} \tag{33}$$

38. The modified Bessel function of zeroth order is evaluated in Subroutine BESI, which was obtained from the NRCC Scientific Subroutine Library. It was verified on several test cases using a Chemical Rubber Company Handbook of Mathematical Sciences (1978).

Rayleigh transition probabilities

39. The fourth step is the evaluation of the Rayleigh transition probabilities P11 and P22 using Equations 13 and 14, respectively. Subroutine RTP sets up the proper lower and upper array element integration limits of the 1-D and 2-D Rayleigh PDF's (i.e. NL and NU, respectively) for the particular cutoff (i.e. threshold) wave height HC selected. For the P11 transition probability, the lower and upper array elements are, respectively,

$$NL = 1$$
$$NU = \frac{HC}{DELH} \tag{34}$$

Similarly, for the P22 transition probability, these limits are

$$NL = \frac{HC}{DELH}$$
$$NU = NH + 1 \tag{35}$$

40. Subroutine RTPI is called by subroutine RTP and calculates either transition probability P11 or P22 given the proper lower NL and upper NU array element integration limit. The discrete 1-D and 2-D Rayleigh PDF's are integrated numerically using Subroutine QSF, obtained from the NRC Scientific Subroutine Library. A dummy 1-D array at equidistant points is evaluated using a combination of Simpson's and Newton's 3/8 rules. It has been thoroughly tested by NRC.

Statistics of run of high waves

41. The fifth step is the calculation of group statistics for a run of high waves based on the transition probability P22. The probabilities for various run lengths PHR are calculated using Equation 25. The mean run length J1M and standard deviation of run length SIGJ1 are given by Equations 26 and 27.

Statistics of total run

42. The final step is the calculation of group statistics for a total run using transition probabilities P11 and P22. The probability distribution

PTR, the mean run length J2M, and the standard deviation of the run length SIGJ2 are defined in Equations 28, 29, and 30, respectively.

Input and Output Variables

Input variables

43. Subroutine INPUT queries the user for the five variables listed in Table 8. Figure 5 is an example of the input required. The value of RHH1 is dimensionless and should be between zero and unity. (See Part III of this report for range of values for typical wave conditions.) If an actual time series of wave height measurements is used, then the parameter NH should be equal to the total number of points in the series. Otherwise, a value of NH

Table 8

Input Variables

Variable	Description
RHH1	Correlation coefficient
NH	Total number of wave height measurements or total number of intervals of dummy wave height variable H1 and/or H2
DELH	Wave height increment between successive H1 or H2 wave heights
HM	Mean wave height
HC	Threshold wave height

```
$ RUN KIMUR5

ENTER 1 FOR TEST CASE:
2

ENTER VALUES FOR:
RHH1     CORRELATION COEFFICIENT
NH       TOTAL # OF WAVE HEIGHT MEASUREMENTS
DELH     WAVE HEIGHT INCREMENT
         UPPER WAVE HGHT INTEGRATION LIMIT = NH * DELH
HM       MEAN WAVE HEIGHT
HC       THRESHOLD WAVE HEIGHT
.68 400 .0025 .33 .33

FORTRAN STOP
```

Figure 5. Example input format for Program KIMUR5

of 400 has been found to give reasonable results. The program is dimensioned for up to 500, however. The third input variable, DELH, corresponds to the width of the class interval in a nondimensionalized histogram or distribution function of wave heights. The smaller this value, the more accurate the results. The product of NH and DELH gives the upper wave height integration limit HU which should be greater than or equal to the largest wave height in the time series. The mean wave height should be determined from the time series if actual wave heights are used. According to Goda (1985), the relationship between the maximum wave height H_{max} and the mean wave height H_m is approximately

$$H_m = 0.31 \text{ to } 0.39\ H_{max} \tag{36}$$

since the significant wave height $H_s = 1.6\ H_m$ and $H_{max} = 1.6$ to $2.0\ H_s$, depending on the number of points in the time series. A value for H_m of 0.33 is representative of a Rayleigh distributed wave height for $H_{max} = 1.0$, assuming the statistically derived maximum wave height is equal to the largest wave in the time series of wave heights. Finally, the threshold wave height can be equal to the mean, median, or significant wave height. Formulas relating these three parameters are

$$\begin{aligned} H_{med} &= 0.939\ H_m \\ H_s &= 1.597\ H_m \end{aligned} \tag{37}$$

Output variables

44. Output variables are written by subroutine OUTPT to a disk file for later viewing or printing. Figure 6 is an example of the output file KIMUR.OUT. The five input variables are listed along with the upper wave height integration limit. The correlation parameter K and the two transition probabilities P11 and P22 are written in the "output variables" section. For a run of high waves, the probabilities PHR of a run of successive high waves of run lengths of 1 through 25 (in 10F7.3 format), the mean group length, J1M, and the standard deviation of the group length, SIGJ1, are given. The first element of PHR corresponds to the probability of a run of length 1, the second element is the probability of a run of 2 waves, the third a run of

```
                              RESULTS FROM PROGRAM KIMUR5
                              GROUP RUN LENGTH STATISTICS

    *****INPUT VARIABLES*****

CORRELATION COEFFICIENT, RHH1 =                       0.6800
TOTAL # OF WAVE HEIGHT MEASUREMENTS, NH =               400
WAVE HEIGHT INCREMENT, DELH =                         0.0025
UPPER WAVE HGHT INTEGRATION LIMIT =                   1.0000
MEAN WAVE HEIGHT, HM =                                0.3300
THRESHOLD WAVE HEIGHT, HC =                           0.3300

    *****OUTPUT VARIABLES*****

CORRELATION PARAMETER, K =                            0.8399
PROBABILITY NEITHER H1 NOR H2 EXCEEDS, P11            0.7617
PROBABILITY BOTH H1 & H2 EXCEED, P22 =                0.7202

*****HIGH WAVE RUN GROUP RESULTS*****
PROBABILITIES, PHR:
 0.280  0.202  0.145  0.105  0.075  0.054  0.039  0.028  0.020  0.015
 0.011  0.008  0.005  0.004  0.003  0.002  0.001  0.001  0.001  0.001
 0.000  0.000  0.000  0.000  0.000

MEAN GROUP LENGTH, J1M =                              3.5743
STD DEV GROUP LENGTH, SIGJ1 =                         3.0334

*****TOTAL WAVE RUN GROUP RESULTS*****
PROBABILITIES, PTR:
 0.000  0.067  0.099  0.110  0.109  0.101  0.090  0.078  0.066  0.055
 0.045  0.037  0.030  0.024  0.019  0.015  0.012  0.010  0.008  0.006
 0.005  0.004  0.003  0.002  0.002

MEAN GROUP LENGTH, J2M =                              7.7715
STD DEV GROUP LENGTH, SIGJ2 =                         4.7561
```

Figure 6. Example output format from Program KIMUR5

3 waves, etc. Similarly, for the total run, the first 25 probabilities PTR of a run of length 1 through 25, the mean length, J2M, and the standard deviation of the run length, SIGJ2, are given.

PART V: PROGRAM VERIFICATION

45. In this section, verification of the program with sample test cases is described and discussed.

Test Case 1

46. The first test case is based on actual wave height data. Goda (1985) describes an example of 97 waves with a mean wave height of 2.1 m. The wave heights are distributed in a range from 0.1 to 5.5 m. Thus, the inputs to Program KIMUR5 are RHH1 = 0.68, NH = 97, DELH = 0.0567 (HU = NH * DELH = 5.5 m), and HM = 2.1. Table 9 summarizes the results for three test cases for each of the mean, median, and significant wave heights as threshold wave height (using Equation 37). The effect of various threshold wave heights on the value of the run length calculated is readily apparent. The calculated values appear to be reasonable when compared with Goda's.

Table 9

Summary of Test Case 1 Results

Threshold Wave Height		Transition Probabilities		Mean Run Lengths	
Description	HC	P11	P22	High	Total
Median	2.0	0.727	0.735	3.775	7.431
Mean	2.1	0.753	0.719	3.554	7.594
Significant	3.3	0.922	0.543	2.186	14.922

Test Case 2

47. Van Vledder (1983b) calculated the transition probabilities P11 and P22 and the mean run lengths J1M and J2M for various values of the correlation coefficient RHH1 and threshold wave heights of the median, mean, and significant wave heights. For a nondimensional maximum wave height of 1.0, the following inputs were used in program KIMUR5: RH1 = .68, NH = 400, DELH = 0.0025, and HM = 0.33. Table 10 shows the comparison between the KIMUR5 calculated values and the van Vledder values (given by van Vledder to four significant places). The average percent error listed is the average of the

Table 10

Summary of Test Case 2 Results

Threshold Wave Height			Transition Probability		Mean Run Length		Average Percent Error
Description	Source	H_c	P11	P22	J1M	J2M	
Median	van Vledder		0.7371	0.7371	3.8037	7.6073	
	KIMUR5	0.31	0.7334	0.7378	3.8146	7.5650	0.08
Mean	van Vledder		0.7651	0.7197	3.5674	7.8243	
	KIMUR5	0.33	0.7617	0.7202	3.5743	7.7715	0.35
Significant	van Vledder		0.9314	0.5592	2.2686	16.8359	
	KIMUR5	0.53	0.9315	0.5525	2.2348	16.8360	0.67

absolute values of the four errors between the two transition probabilities and the two mean run lengths. These errors are defined as the difference between van Vledder's and the calculated values for each quantity divided by van Vledder's value. Thus, the average percent error is less than 0.67 percent for all cases tested and shows very good agreement with van Vledder's results.

48. Van Vledder* recommends that the total number of increments NH should be greater than 150 to 200. Table 11 lists the differences in calculated values for a threshold wave height equal to the mean wave height for various NH values of 50, 100, 200, 400, and 500. The average percent error between the program's values and van Vledder's decreases markedly for increases in the number of intervals.

Discussion of Results

49. The agreement of the KIMUR5 model with van Vledder's calculated values is excellent. Many factors could account for the slight differences observed. According to van Vledder,* his program is dimensioned for double precision, explicitly calculates the 1-D Rayleigh integral, and uses a

* Personal Communication, 2 January 1986, with Dr. G. Ph. van Vledder, Delft University of Technology, Department of Civil Engineering, Delft, The Netherlands.

Table 11

Effect of Various NH Values

NH	Transition Probabilities		Mean Run Lengths		Average Percent Error
	P11	P22	High	Total	
van Vledder	0.7651	0.7197	3.5674	7.8243	--
50	0.7219	0.7445	3.9141	7.5105	5.70
100	0.7514	0.7268	3.6605	7.6837	1.79
200	0.7584	0.7224	3.6025	7.7408	0.83
400	0.7617	0.7202	3.5743	7.7715	0.35
500	0.7624	0.7198	3.5687	7.7778	0.25

maximum wave height value of 10 times the mean wave height as the upper wave height integration limit. His program also iteratively checks for the optimum number of steps to use in calculating the values of the joint Rayleigh integral.

50. My investigations showed that double precision did not make any noticeable difference (to E-04 precision for the output results) for a value of NH of 100. Different programming techniques, the numerical integration routine used, explicit calculation of the 1-D Rayleigh integral, and the method of selection of the lower and upper cutoff limits in the integrals probably account for the slight differences observed.

PART VI: SUMMARY AND RECOMMENDATIONS

51. The coastal engineering research community has recognized the need to model wave groups as well as spectral waves. High waves in groups can produce more damage than isolated high waves, and engineers are finding that this groupiness has important ramifications in the motions and resonances of moored structures and vessels, harbor resonance, stability and overtopping of shore protection structures, and surf beat. Wave grouping is especially significant in shallow-water laboratory basins such as the CERC directional spectral wave basin. The control, prediction, measurement, and analysis of wave groups are necessary for CERC to fulfill its mission of advancing the state of the art in coastal engineering and laboratory physical modeling.

52. Successive wave heights are dependent phenomena. Thus, the Kimura model is a better predictor of run lengths than the Goda model. The computer program KIMUR5 gives excellent agreement with van Vledder's values. Additional development and testing with simulated and measured data might lead to better agreement.

53. Future enhancements might include converting the main program to a subroutine so that it can be called from a SIWEH analysis and/or spectral analysis program. Presently the correlation coefficient is an input parameter. An option could be to calculate it directly, from the wave height time-history using an autocorrelation procedure, or spectrally, using Battjes' or Goda's method. Finally, the Rayleigh PDF's are calculated using the mean wave height. An option to allow the use of other wave height values, such as median and significant wave heights, could be included.

54. Correlation among the spectral peakedness parameter, the correlation coefficient, and the groupiness factor obtained from a SIWEH time-history could be investigated further. Also, additional research into the relationship between the groupiness factor and the degree of grouping in simulated data and wind-generated waves would be beneficial to increase our understanding into wave grouping physics.

55. An analogous program could be developed for the groupiness statistics of successive wave periods which fall within a certain period band. Usually, values between 0.7 to 1.2 times the mean wave period are most important. The development for wave periods again assumes the time series of periods is mutually correlated and forms a Markov Chain. Kimura (1980) showed

that the Weibull distribution would replace the corresponding one- and two-dimensional Rayleigh distributions. The spectral width parameter was the analogous spectral parameter (i.e. spectral peakedness for heights) most closely associated with the correlation coefficient for wave periods.

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APPENDIX A: LISTING OF PROGRAM KIMUR5

```
      PROGRAM KIMUR5
C     SINGLE PRECISION, MANUAL INPUT, DIMENSIONED FOR 501 ARRAY SIZE
C
C     PROGRAM TO CALCULATE GROUP RUN LENGTH STATISTICS BASED ON KIMURA'S
C     METHOD. THE TIME SERIES OF WAVE HEIGHTS ARE ASSUMED TO BE MUTTUALLY
C     CORRELATED AND FORM A MARKOV CHAIN.  TRANSITION PROBABILITIES ARE
C     DETERMINED FROM THE TWO-DIMENSIONAL RAYLEIGH JOINT PROBABILITY DENSITY
C     FUNCTION FOR SUCCESSIVE WAVES.  FOR BOTH A RUN OF HIGH WAVES AND A
C     TOTAL RUN, THE PROBABILITIES OF RUNS OF DIFFERENT LENGTHS, THE AVERAGE
C     RUN LENGTH, AND THE STANDARD DEVIATION OF THE RUN LENGTH ARE CALCULATE
C
      REAL G(501),PHR(25),PTR(25)
      REAL K,J1M,J2M
      REAL P(501,501)
      DATA RHH1,NH,DELH,HM,HC/.681,97,.0567,2.1,2.1/
C
C     OPEN DISK FILE FOR OUTPUT
C
      OPEN(UNIT=2,FILE='KIMUR.OUT',STATUS='UNKNOWN')
C
C     QUERY USER FOR TEST CASE
C
      WRITE(6,10)
 10   FORMAT(/,' ENTER 1 FOR TEST CASE: ')
      READ(5,*) IASK
      IF(IASK .EQ. 1) GO TO 20
C
C     INPUT PARAMETERS
C
      CALL INPUT(RHH1,NH,DELH,HM,HC)
C     CALCULATE KAPPA CORRELATION PARAMETER
C
 20   CONTINUE
      CALL KAPPA(RHH1,K)
C
C     CALCULATE RAYLEIGH PROBABILITY DENSITY FUNCTION G(H1)
C
      CALL RPDF(NH,DELH,HM,G)
C
C     CALCULATE RAYLEIGH JOINT PROBABILITY DENSITY FUNCTION P(H1,H2)
C
      CALL RJPDF(NH,DELH,HM,K,P)
C
C     CALCULATE RAYLEIGH TRANSITION PROBABILITIES P11 & P22
C
      CALL RTP(NH,DELH,HC,G,P,P11,P22)
C
C     CALCULATE HIGH RUN STATISTICS
C
      CALL HRUN(P22,PHR,J1M,SIGJ1)
C
C     CALCULATE TOTAL RUN STATISTICS
C
      CALL TRUN(P11,P22,PTR,J2M,SIGJ2)
C
C     OUTPUT RESULTS TO TERMINAL OR LINE PRINTER
C
      CALL OUTPT(RHH1,NH,DELH,HM,HC,K,G,P,P11,P22,PHR,J1M,SIGJ1,
     &             PTR,J2M,SIGJ2)
C
```

```
      STOP
      END
C
      SUBROUTINE INPUT(RHH1,NH,DELH,HM,HC)
C
C     QUERIES USER FOR INPUT PARAMETERS FOR KIMURA'S GROUP LENGTH PROGRAM
C
      WRITE(6,10)
 10   FORMAT(/,' ENTER VALUES FOR: ',
     &        /,' RHH1    CORRELATION COEFFICIENT ',
     &        /,' NH      TOTAL # OF WAVE HEIGHT MEASUREMENTS ',
     &        /,' DELH    WAVE HEIGHT INCREMENT ',
     &        /,'         UPPER WAVE HGHT INTEGRATION LIMIT = NH * DELH',
     &        /,' HM      MEAN WAVE HEIGHT ',
     &        /,' HC      THRESHOLD WAVE HEIGHT ')
      READ(5,*) RHH1,NH,DELH,HM,HC
C
      RETURN
      END
C
      SUBROUTINE KAPPA(RHH1,K)
C
C     CALCULATES CORRELATION PARAMETER K (KAPPA=2*RHO) GIVEN CORRELATION
C     COEFFICIENT RHH1 (ALSO KNOWN AS GAMMA(H)) USING SERIES APPROXIMATION
C     METHOD OF BATTJES FOR THE ELLIPTIC INTEGRALS OF THE 1ST & 2ND KIND.
C
      REAL K,K2
C
C     INITIALIZE PARAMETERS
C
      PI = 4. * ATAN(1.0)
      C = (16. - 4. * PI) / PI
C
C     KAPPA K
C
      R = C * RHH1
      R2 = R * R
      R3 = R * R2
      K2 = R - R2/16. - R3/128.
      K = SQRT(K2)
C
      RETURN
      END
C
      SUBROUTINE RPDF(NH,DELH,HM,G)
C
C     CALCULATES 1-D RAYLEIGH PROBABILITY DENSITY FUNCTION G(H1).
C
C     INPUT VARIABLES
C     NH      = # OF INTERVALS OF DUMMY WAVE HEIGHT VARIABLE H1
C     DELH    = DELTA INCREMENT BETWEEN SUCCESSIVE H1 WAVE HEIGHTS
C     HM      = MEAN WAVE HEIGHT
C
C     OUTPUT VARIABLES
C     G       = 1-D RAYLEIGH PROBABILITY DENSITY FUNCTION G(H1)
C
      REAL G(501)
C
C     INITIALIZE CONSTANTS
C
```

```
      PIO2 = 2. * ATAN(1.0)
      PIO4 = PIO2 / 2.
      HM2 = HM * HM
      Q1 = PIO2 / HM2
      Q2 = PIO4 / HM2
      NHP1 = NH + 1
C
C     RAYLEIGH PDF
C
      DO     I=1,NHP1
        H1 = (I-1) * DELH
        H12 = H1 * H1
        G(I) = (Q1 * H1) * EXP(- Q2 * H12)
      END DO
C
      RETURN
      END
C
      SUBROUTINE RJPDF(NH,DELH,HM,K,P)
C
C     CALCULATES 2-D RAYLEIGH JOINT PROBABILITY DENSITY FUNCTION P(H1,H2)
C
C     INPUT VARIABLES
C     NH      = TOTAL # WAVE HEIGHT INTERVALS OF DUMMY VARIABLE H1 & H2
C     DELH    = DELTA INCREMENT BETWEEN SUCCESSIVE WAVE HEIGHTS
C     HM      = MEAN WAVE HEIGHT
C     K       = CORRELATION PARAMETER KAPPA
C
C     OUTPUT VARIABLES
C     P       = RAYLEIGH JOINT PDF P(H1,H2)
C
      DOUBLE PRECISION DX,BI(1)
      REAL P(501,501)
      REAL K
C
C     INITIALIZE PARAMETERS
      PIO4 = ATAN(1.0)
      PIO2 = 2. * PIO4
      PI = 4. * PIO4
      PI2O4 = PI * PIO4
      HM2 = HM * HM
      HM4 = HM2 * HM2
      CK2 = 1.0 - K * K
      A1 = PI2O4 / (HM4 * CK2)
      B1 = PIO4 / (HM2 * CK2)
      X1 = (PIO2 * K) / (HM2 * CK2)
      NHP1 = NH + 1
C
C     RAYLEIGH JOINT PROBABILITY DENSITY FUNCTION, P(H1,H2) = A * B * C
C
      DO     I=1,NHP1
C       DUMMY H1 WAVE HEIGHT VARIABLE
        H1 = (I-1) * DELH
        H12 = H1 * H1
        DO     J=1,NHP1
C         DUMMY H2 WAVE HEIGHT VARIABLE
          H2 = (J-1) * DELH
          H22 = H2 * H2
          H1H2 = H1 * H2
C     A FACTOR
```

```
         A = A1 * H1H2
C        B FACTOR
         B = EXP(- B1 * (H12 + H22))
C        X ARGUMENT FOR MODIFIED BESSEL FUNCTION OF ZERO ORDER
         X = X1 * H1H2
         DX = DBLE(X)
C        MODIFIED BESSEL FUNCTION OF ZERO ORDER
         CALL BESI(0,DX,BI,1)
C        C FACTOR
         C = EXP(X) * BI(1)
 5       FORMAT(2I5,6F10.3)
C        RAYLEIGH JOINT PDF
         P(I,J) = A * B * C
C       WRITE(6,5) I,J,A,B,X,BI(1),C,P(I,J)
       END DO
     END DO
C
     RETURN
     END
C
     SUBROUTINE BESI(N,X,BI,LOG)
C    VERSION 1.0 - OF DEC. 1984
C
C*************************************************************************
C*************************************************************************
C***************                          ***************************
C***************       SUBROUTINE BESI    ***************************
C***************                          ***************************
C*************************************************************************
C*************************************************************************
C*                                                                       *
C------------------------------------------------------------------------*
C*   SUBROUTINE SUMMARY                                                  *
C------------------------------------------------------------------------*
C*                                                                       *
C*   CALCULATES THE VALUE OF THE BESSEL FUNCTION I, DEXP(-X)*I(K,X).     *
C*                                                                       *
C------------------------------------------------------------------------*
C*   SUBROUTINE DESCRIPTION                                              *
C------------------------------------------------------------------------*
C*                                                                       *
C*   THIS SUBROUTINE CALCULATES THE VALUE OF THE FUNCTION                *
C*   DEXP(-X)*I(K,X), WHERE I IS THE BESSEL FUNCTION OF NON NEGATIVE     *
C*   INTEGRAL ORDER K=0,N, AND NON NEGATIVE REAL ARGUMENT X. ALL         *
C*   CALCULATIONS ARE IN DOUBLE PRECISION.                               *
C*                                                                       *
C------------------------------------------------------------------------*
C*   INPUT PARAMETERS:                                                   *
C------------------------------------------------------------------------*
C*                                                                       *
C*   N    -  THE ORDER OF THE BESSEL FUNCTION I (N=0)                    *
C*   X    -  THE ARGUMENT OF THE BESSEL FUNCTION I (X=0)                 *
C*                                                                       *
C------------------------------------------------------------------------*
C*   OUTPUT PARAMETERS:                                                  *
C------------------------------------------------------------------------*
C*                                                                       *
C*   BI  - A ONE DIMENSIONAL ARRAY CONTAINING THE VALUE OF THE           *
C*         FUNCTION DEXP(-X)*I(K,X) IN ITS K+1 ELEMENT FOR K=0,N.        *
C*         THE DIMENSION OF BI IN THE CALLING PROGRAM MUST BE AT         *
```

```
C*          LEAST N+1.                                                      *
C*                                                                          *
C*                                                                          *
C*--------------------------------------------------------------------------*
C*    SUBROUTINES AND FUNCTIONS CALLED:                                     *
C*--------------------------------------------------------------------------*
C*                                                                          *
C*    NAME                          DESCRIPTION                             *
C*                                                                          *
C*--------------------------------------------------------------------------*
C*    ERROR CODES:                                                          *
C*--------------------------------------------------------------------------*
C*                                                                          *
C*    AN APPROPRIATE MESSAGE IS WRITTEN TO THE LOG WHEN X<0 OR N<0.         *
C*                                                                          *
C*--------------------------------------------------------------------------*
C*                                                                          *
C*    SUBROUTINE CREATION DATE AND AUTHOR:                                  *
C*        VERSION 1.5 - 01 SEPT 1970                                        *
C*                    - NRC COMPUTATION CENTRE                              *
C*                                                                          *
C*--------------------------------------------------------------------------*
C*                                                                          *
C*    SUBROUTINE MODIFICATIONS:                                             *
C*                                                                          *
C*    VERSION               DATE                   AUTHOR/FIRM              *
C*      1.5             01 SEPT 1970       NRC COMPUTATION CENTRE           *
C*                                                                          *
C*    DESCRIPTION:  ORIGINAL                                                *
C*                                                                          *
C*      1.6             06 DEC. 1984       GORDON FOUGERE/EMR               *
C*                                                                          *
C*    DESCRIPTION:  THE SUBROUTINE WAS UPDATED TO REFLECT CURRENT           *
C*                  GEDAP PROGRAM STANDARDS.                                *
C*                                                                          *
C***************************************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL T,SQRT
      DIMENSION BI(1)
      DATA DMIN/0.5397605346934D-37/,AMAX/1.8526734277970+18/
C
C     AMAX*DMIN=0.9999999999998D-19
C
C     CHECK THE ARGUMENTS
C
      IF(N.GE.0) GO TO 1
      WRITE(LOG,100) N,X
  100 FORMAT(/10X,'BESI NEGATIVE ARGUMENT. ORDER=',I11,' ARGUMENT=',
     A D23.16//)
      BI(1)=0
      RETURN
    1 N1=N+1
      IF(X.GT.1.D-6) GO TO 6
      IF(X.GE.0.D0) GO TO 3
      WRITE(LOG,100) N,X
      DO 2 I=1,N1
    2 BI(I)=0
      RETURN
C
```

```
C       I(K,X) = (X/2)**K/(K FACTORIAL)'FOR X <= 1.0D-8
C       DEXP(-X) = 1.D0-X FOR X <= 1.0D-8
C
      3 BI(1)=1.D0-X
        IF(N.EQ.0) RETURN
        ISW=0
        IF(X.EQ.0.D0) ISW=1
        A=0.D0
        IF(ISW.EQ.0) A=2.D0*DMIN/X
        DO 5 I=1,N
        BI(I+1)=0.D0
        IF(ISW.EQ.1) GO TO 5
        IF(BI(I).GT.A*I) GO TO 4
        ISW=1
        GO TO 5
      4 BI(I+1)=BI(I)*X/(2*I)
      5 CONTINUE
        RETURN
C
C       CALCULATE STARTING POINT FOR BACKWARD RECURRENCE
C
      6 T=X
        SQROOT=SQRT(T)
        N2=M1
        IF(T.GT.20.0) GO TO 7
        M=10.55*T-8.11*ABS(T-0.23)-1.23*ABS(T-1.73)-0.47*ABS(T-6.78)+14.11
        GO TO 8
      7 M=SQROOT*(-0.8310185E-2*ABS(T-15.06499)+0.6837897E-2*
       A ABS(T-44.82328)+0.1397544E-2*ABS(T-131.9504)+0.8336472E-4*
       B ABS(T-847.8685)+0.3283161E-6*ABS(T-23001.16)+8.9806)+1.0
      8 IF(M.GE.N) GO TO 12
        M=N
        IF(T.GT.14.5) GO TO 9
        MMAX=24716.33*T-24107.58*ABS(T-0.105E-3)-536.05*ABS(T-0.514E-2)-
       A 50.16*ABS(T-0.051)-13.69*ABS(T-0.25)-3.97*ABS(T-0.95)-1.77*
       B ABS(T-2.55)-0.86*ABS(T-8.80)+41.5
        GO TO 10
      9 MMAX=SQROOT*(-0.6479355E-1*(T-14.40017)+0.5534451E-1*
       A ABS(T-44.78195)+0.8671853E-2*ABS(T-167.343)+0.7717749E-3*
       B ABS(T-832.274)+0.5685193E-5*ABS(T-35155.43)+0.1244541E-6*
       C ABS(T-288568.5)+22.4775)+1.0
     10 IF(MMAX.GT.N) GO TO 12
        DO 11 I=MMAX,N
     11 BI(I+1)=0.D0
        M=MMAX-1
        N2=MMAX
     12 M1=M-1
C
C       CALCULATE THE RATIO I(M,X)/I(M-1,X)
C
        L=2.0*SQROOT+6.0
        RATIO=0.D0
        DO 13 I=1,L
        FLOT=2*(M+L-I)
        RATIO= X/(FLOT+X*RATIO)
     13 CONTINUE
C
C       COMPUTE F(M),F(M-1),.....,F(0), AND ALPHA
C
        A=1.E-19
```

```
      XX=2.D0/X
      FM2=A
      FM1=A/RATIO
      IF(M.GT.N) GO TO 14
      BI(M+1)=FM2
      BI(M)=FM1
      GO TO 15
   14 IF(M1.EQ.N) BI(N1)=FM1
   15 ALPHA=FM1+FM2
      DO 16 I=1,M1
      MI=M-I
      FM0=MI*XX*FM1+FM2
      IF(MI.LE.N1) BI(MI)=FM0
      ALPHA=ALPHA+FM0
      FM2=FM1
   16 FM1=FM0
      ALPHA=2.D0*ALPHA-FM0
C
C     CALCULATE THE VALUES OF DEXP(-X)*I(K,X),K=0,N.
C
      IF(ALPHA.LE.AMAX) GO TO 18
      A=DMIN*ALPHA
   17 IF(BI(N2).GT.A) GO TO 18
      BI(N2)=0.D0
      N2=N2-1
      GO TO 17
   18 ALPHA=1.D0/ALPHA
      DO 19 I=1,N2
   19 BI(I)=BI(I)*ALPHA
      RETURN
      END
C
      SUBROUTINE RTP(NH,DELH,HC,G,P,P11,P22)
C
C     CALCULATES RAYLEIGH TRANSITION PROBABILITIES P11 & P22.
C
C     INPUT VARIABLES
C     NH      = # OF INTERVALS OF DUMMY WAVE HEIGHT VARIABLE H1 & H2
C     DELH    = DELTA INCREMENT BETWEEN SUCCESSIVE H1 & H2 WAVE HEIGHTS
C     HC      = CUTOFF OR THRESHOLD WAVE HEIGHT, ALSO H*
C     G       = RAYLEIGH 1-D PROBABILITY DENSITY FUNCTION G(H1)
C     P       = RAYLEIGH 2-D JOINT PROBABILITY DENSITY FUNCTION P(H1,H2)
C
C     OUTPUT VARIABLES
C     P11     = TRANSITION PROBABILITY, NEITHER H1 NOR H2 EXCEEDS THRESHOLD
C                 WAVE HEIGHT HC
C     P22     = TRANSITION PROBABILITY, BOTH H1 & H2 EXCEED THRESHOLD WAVE
C                 WAVE HEIGHT HC
C
      REAL G(501)
      REAL P(501,501)
C
C     P11 TRANSITION PROBABILITY
C
      NL = 1
      NU = HC / DELH
      CALL RTPI(NL,NU,DELH,G,P,P11)
C
C     P22 TRANSITION PROBABILITY
C
```

```
      NL = HC /DELH
      NU = NH + 1
      CALL RTPI(NL,NU,DELH,Q,P,P22)
C
      RETURN
      END
C
      SUBROUTINE RTPI(NL,NU,DELH,Q,P,PROB)
C
C     INTEGRATES 1-D & 2-D RAYLEIGH PROBABILITY DENSITY FUNCTIONS
C     Q(H1) & P(H1,H2)
C
C     INPUT VARIABLES
C     NL     = LOWER INTEGRATION LIMIT ARRAY ELEMENT
C     NU     = UPPER INTEGRATION LIMIT ARRAY ELEMENT
C     DELH   = DELTA INCREMENT BETWEEN SUCCESSIVE H1 OR H2 WAVE HEIGHTS
C     P      = RAYLEIGH 2-D JOINT PROBABILITY DENSITY FUNCTION P(H1,H2)
C     Q      = RAYLEIGH 1-D PROBABILITY DENSITY FUNCTION Q(H1)
C
C     OUTPUT VARIABLES
C     PROB   = TRANSITION PROBABILITY, EITHER P11 OR P22
C
      REAL P(501,501)
      REAL Q(501),PH1(501),PH2(501),Z(501)
C
C     2-D RAYLEIGH JOINT PROBABILITY DENSITY FUNCTION INTEGRAL
C     IN NUMERATOR
C
      M = 0
C     SHIFT PDF TO DUMMY ARRAY PH2
      DO   I=NL,NU
        N = 0
        DO   J=NL,NU
          N = N + 1
          PH2(N) = P(I,J)
        END DO
C       INTEGRATE IN H2 DIRECTION & CREATE NEW DUMMY ARRAY PH1
        NDIM = N
        CALL QSF(DELH,PH2,Z,NDIM)
        M = M + 1
        PH1(M) = Z(NDIM)
      END DO
C     INTEGRATE IN H1 DIRECTION USING DUMMY ARRAY PH1
      NDIM = M
      CALL QSF(DELH,PH1,Z,NDIM)
      SUMN = Z(NDIM)
C
C     1-D RAYLEIGH PROBABILITY DENSITY FUNCTION INTEGRAL IN DENOMINATOR
C
C     SHIFT PDF TO DUMMY ARRAY PH1
      N = 0
      DO  I=NL,NU
        N = N + 1
        PH1(N) = Q(I)
      END DO
C     INTEGRATE IN H1 DIRECTION USING DUMMY ARRAY PH1
      NDIM = N
      CALL QSF(DELH,PH1,Z,NDIM)
      SUMD = Z(NDIM)
C
```

```
C     TRANSITION PROBABILITY
C
      PROB = SUMN / SUMD
C
      RETURN
      END
C
      SUBROUTINE QSF(H,Y,Z,NDIM)
C     ..................................................................
C
C        SUBROUTINE QSF
C
C        PURPOSE
C           TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN
C           EQUIDISTANT TABLE OF FUNCTION VALUES.
C
C        USAGE
C          CALL QSF (H,Y,Z,NDIM)
C
C        DESCRIPTION OF PARAMETERS
C           H      - THE INCREMENT OF ARGUMENT VALUES.
C           Y      - THE INPUT VECTOR OF FUNCTION VALUES.
C           Z      - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE
C                    IDENTICAL WITH Y.
C           NDIM   - THE DIMENSION OF VECTORS Y AND Z.
C
C        REMARKS
C           NO ACTION IN CASE NDIM LESS THAN 3.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           BEGINNING WITH Z(1)=0, EVALUATION OF VECTOR Z IS DONE BY
C           MEANS OF SIMPSONS RULE TOGETHER WITH NEWTONS 3/8 RULE OR A
C           COMBINATION OF THESE TWO RULES. TRUNCATION ERROR IS OF
C           ORDER H**5 (I.E. FOURTH ORDER METHOD). ONLY IN CASE NDIM=3
C           TRUNCATION ERROR OF Z(2) IS OF ORDER H**4.
C           FOR REFERENCE, SEE
C           (1) F.B.HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS,
C               MCGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP.71-76.
C           (2) R.ZURMUEHL, PRAKTISCHE MATHEMATIK FUER INGENIEURE UND
C               PHYSIKER, SPRINGER, BERLIN/GOETTINGEN/HEIDELBERG, 1963,
C               PP.214-221.
C
C     ..................................................................
C
      DIMENSION Y(1),Z(1)
C
      HT=.3333333*H
      IF(NDIM-5)7,8,1
C
C     NDIM IS GREATER THAN 5. PREPARATIONS OF INTEGRATION LOOP
    1 SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
      AUX2=HT*(Y(1)+3.875*(Y(2)+Y(5))+2.625*(Y(3)+Y(4))+Y(6))
```

```
      SUM2=Y(5)+Y(5)
      SUM2=SUM2+SUM2
      SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
      Z(1)=0.
      AUX=Y(3)+Y(3)
      AUX=AUX+AUX
      Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
      Z(3)=SUM1
      Z(4)=SUM2
      IF(NDIM-6)5,5,2
C
C     INTEGRATION LOOP
    2 DO 4 I=7,NDIM,2
      SUM1=AUX1
      SUM2=AUX2
      AUX1=Y(I-1)+Y(I-1)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
      Z(I-2)=SUM1
      IF(I-NDIM)3,6,6
    3 AUX2=Y(I)+Y(I)
      AUX2=AUX2+AUX2
      AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
    4 Z(I-1)=SUM2
    5 Z(NDIM-1)=AUX1
      Z(NDIM)=AUX2
      RETURN
    6 Z(NDIM-1)=SUM2
      Z(NDIM)=AUX1
      RETURN
C
C     END OF INTEGRATION LOOP
C
    7 IF(NDIM-3)12,11,8
C
C     NDIM IS EQUAL TO 4 OR 5
    8 SUM2=1.125*HT*(Y(1)+Y(2)+Y(2)+Y(2)+Y(3)+Y(3)+Y(3)+Y(4))
      SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      Z(1)=0.
      AUX1=Y(3)+Y(3)
      AUX1=AUX1+AUX1
      Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
      IF(NDIM-5)10,9,9
    9 AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      Z(5)=SUM1+HT*(Y(3)+AUX1+Y(5))
   10 Z(3)=SUM1
      Z(4)=SUM2
      RETURN
C
C     NDIM IS EQUAL TO 3
   11 SUM1=HT*(1.25*Y(1)+Y(2)+Y(2)-.25*Y(3))
      SUM2=Y(2)+Y(2)
      SUM2=SUM2+SUM2
      Z(3)=HT*(Y(1)+SUM2+Y(3))
      Z(1)=0.
      Z(2)=SUM1
   12 RETURN
```

```
      END
C
      SUBROUTINE HRUN(P22,PHR,J1M,SIGJ1)
C
C     CALCULATES HIGH RUN GROUP STATISTICS OF PROBABILITY FOR DIFFERENT
C     RUN LENGTHS, MEAN RUN LENGTH, & STANDARD DEVIATION OF RUN LENGTH.
C
C     INPUT VARIABLES
C     P22     = TRANSITION PROBABILITY FOR SIMULTANEOUS EXCEEDANCE OF
C                THRESHOLD BY BOTH H1 & H2 WAVE HEIGHTS
C
C     OUTPUT VARIABLES
C     PHR     = PROBABILITY OF RUN LENGTH HAVING LENGTH OF J1, F(J1)
C     J1M     = MEAN RUN LENGTH
C     SIGJ1   = STANDARD DEVIATION OF RUN LENGTH
C
      REAL J1M
      REAL PHR(25)
C
C     PROBABILITY OF RUN OF LENGTH J1, PHR(J1)
C
      IF(P22 .EQ. 0.) P22=1.E-05
      C1P22 = 1.0 - P22
      DO    J1=1,25
        PHR(J1) = P22**(J1-1) * C1P22
      END DO
C
C     MEAN RUN LENGTH
C
      J1M = 1. / C1P22
C
C     STANDARD DEVIATION OF RUN LENGTH
C
      SIGJ1 = SQRT(P22) / C1P22
C
      RETURN
      END
C
      SUBROUTINE TRUN(P11,P22,PTR,J2M,SIGJ2)
C
C     CALCULATES TOTAL RUN GROUP STATISTICS OF PROBABILITY FOR DIFFERENT
C     RUN LENGTHS, MEAN RUN LENGTH, & STANDARD DEVIATION OF RUN LENGTH.
C
C     INPUT VARIABLES
C     P11     = TRANSITION PROBABILITY FOR NEITHER H1 NOR H2 EXCEEDING
C                THRESHOLD WAVE HEIGHT HC
C     P22     = TRANSITION PROBABILITY FOR SIMULTANEOUS EXCEEDANCE OF
C                THRESHOLD HC BY BOTH H1 & H2 WAVE HEIGHTS
C
C     OUTPUT VARIABLES
C     PTR     = PROBABILITY OF RUN LENGTH HAVING LENGTH OF J2, F(J2)
C     J2M     = MEAN RUN LENGTH
C     SIGJ2   = STANDARD DEVIATION OF RUN LENGTH
C
      REAL J2M
      REAL PTR(25)
C
C     PROBABILITY OF RUN OF LENGTH J2, PTR(J2)
C
      IF(P11 .EQ. 0.) P11=1.E-04
```

```
      IF(P22 .EQ. 0.) P22=1.E-05
      C1 = 1.0 - P11
      C2 = 1.0 - P22
      C3 = C1 * C2 / (P11 - P22)
      DO  J2=1,25
        PTR(J2) = C3 * (P11**(J2-1) - P22**(J2-1))
      END DO
C
C     MEAN RUN LENGTH
C
      J2M = 1. / C1 + 1. / C2
C
C     STANDARD DEVIATION OF RUN LENGTH
C
      C12 = C1 * C1
      C22 = C2 * C2
      SIGJ2 = SQRT(P22/C22 + P11/C12)
C
      RETURN
      END
C
      SUBROUTINE OUTPT(RHH1,NH,DELH,HM,HC,K,G,P,P11,P22,PHR,J1M,
     *                 SIGJ1,PTR,J2M,SIGJ2)
C
C     OUTPUTS RESULTS FROM KIMURA'S ALGORITHM FOR CALCULATING GROUP
C     LENGTH STATISTICS BY ASSUMING THAT WAVE HEIGHTS ARE CORRELATED.
C
      REAL P(501,501)
      REAL G(501),PHR(25),PTR(25)
      REAL K,J1M,J2M
C
C     DESCRIPTIVE TITLE
      WRITE(2,10)
 10   FORMAT(1H1,/,T30,'RESULTS FROM PROGRAM KIMURA',
     &            /,T30,'GROUP RUN LENGTH STATISTICS')
C
C     INPUT VARIABLES
C
      HU = NH * DELH
      NHP1 = NH + 1
      WRITE(2,20) RHH1,NH,DELH,HU,HM,HC
 20   FORMAT(//,'      *****INPUT VARIABLES******',
     &       //,' CORRELATION COEFFICIENT, RHH1 =',T45,F10.4,
     &        /,' TOTAL # OF WAVE HEIGHT MEASUREMENTS, NH =',T45,I10,
     &        /,' WAVE HEIGHT INCREMENT, DELH =',T45,F10.4,
     &        /,' UPPER WAVE HGHT INTEGRATION LIMIT =',T45,F10.4,
     &        /,' MEAN WAVE HEIGHT, HM =',T45,F10.4,
     &        /,' THRESHOLD WAVE HEIGHT, HC =',T45,F10.4)
C
C     OUTPUT VARIABLES
C
      WRITE(2,30) K,P11,P22
 30   FORMAT(//,'      *****OUTPUT VARIABLES******',
     &       //,' CORRELATION PARAMETER, K =',T45,F10.4,
     &        /,' PROBABILITY NEITHER H1 NOR H2 EXCEEDS, P11 =',T45,F10.4,
     &        /,' PROBABILITY BOTH H1 & H2 EXCEEDS, P22 =',T45,F10.4)
C     WRITE(2,35)
 35   FORMAT(/,' RAYLEIGH 1-D PDF VALUES ARE:')
C     WRITE(2,40) (G(I),I=1,NHP1)
 40   FORMAT(10F7.3)
```

```
C     WRITE(2,45)
 45   FORMAT(/,' RAYLEIGH 2-D PDF VALUES ARE:')
C     WRITE(2,40) ((P(I,J),I=1,NHP1),J=1,NHP1)
      WRITE(2,50)
 50   FORMAT(//,' *****HIGH WAVE RUN GROUP RESULTS*****',
     &        /,' PROBABILITIES, PHR:')
      WRITE(2,40) (PHR(I),I=1,25)
      WRITE(2,60) J1M,SIGJ1
 60   FORMAT(/,' MEAN GROUP LENGTH, J1M =',T45,F10.4,
     &        /,' STD DEV GROUP LENGTH, SIGJ1 =',T45,F10.4)
      WRITE(2,70)
 70   FORMAT(//,' *****TOTAL WAVE RUN GROUP RESULTS*****',
     &        /,' PROBABILITIES, PTR:')
      WRITE(2,40) (PTR(I),I=1,25)
      WRITE(2,80) J2M,SIGJ2
 80   FORMAT(/,' MEAN GROUP LENGTH, J2M =',T45,F10.4,
     &        /,' STD DEV GROUP LENGTH, SIGJ2 =',T45,F10.4)
C
      RETURN
      END
```

APPENDIX B: LIST OF SYMBOLS USED IN PROGRAM KIMUR5

Symbol	Description
DELH	Delta wave height increment between successive H1 or H2 wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
H	Increment of argument values (i.e. X-array) for calculating integral
HM	Mean wave height
HC	Cutoff or threshold wave height
HU	Upper wave height integration limit HU = NH * DELH
H1	Dummy wave height variable
H2	Dummy wave height variable
J1M	Mean run length for run of high waves
J2M	Mean run length for total run
K	Correlation parameter
NH	Total number of wave height measurements or intervals of dummy wave height parameters H1 or H2 between zero and upper wave height HU in transition probabilities
NL	Lower integration limit array element
NU	Upper integration limit array element
P(H1,H2)	2-D Rayleigh joint probability density function for successive wave heights
PHR(J1)	Probability of run length having length of J1 for run of high waves
PI	3.14159...
PROB	Dummy transition probability, either P11 or P22
PTR(J2)	Probability of run length having length of J2 for total run
P11	Transition probability, neither H1 nor H2 exceeds threshold wave height HC
P22	Transition probability, both H1 and H2 successive wave heights exceed threshold wave height HC
Q(H1)	1-D Rayleigh probability density function for individual wave heights
RHH1	Correlation coefficient
SIGJ1	Standard deviation of run length for run of high waves
SIGJ2	Standard deviation of run length for total run
Y	Function values (i.e. Y-array) to be integrated
Z	Vector array of integrated values

APPENDIX C: DESCRIPTION OF SUBROUTINES

Subroutine BESI

Description:		Calculates value of Bessel Function I, Dexp(-X) * I(K,X) where I is Bessel function of non-negative integral order K=0, N and non-negative real argument X. All calculations are in double precision. National Research Council of Canada (NRCC) Scientific Library Subroutine.
Calling Statement:		SUBROUTINE BESI (N,X,BI,LOG)
Arguments:		
N	I*4	Order of Bessel Function I, N = 0 for zero order (used here)
X	R*8	Argument of Bessel Function I
BI	R*8	I-D array containing value of function Dexp(-X) * I(K,X) in its K+1 element for K=0,N. Dimension of BI in calling program must be at least N+1, equals 1 for N=0 for modified zeroth order
LOG	I*2	Logical unit number of error message when X<0 or N<0

Called By: Subroutine RJPDF
Calls To: None
Reference: NRCC Computation Center

Subroutine HRUN

Description:		Calculates high run group statistics of probabilities for different run lengths, mean run length, and standard deviation of run length.
Calling Statement:		SUBROUTINE HRUN (P22,PHR,J1M,SIGJ1)
Arguments:		
P22	R*4	Transition probability, both H1 and H2 successive wave heights exceed threshold wave height HC
PHR	R*4	Probability of run length having length of J1, P1(J)
J1M	R*4	Mean run length for a run of high waves
SIGJ1	R*4	Standard deviation of run of length J1 for run of high waves

Called By: Program KIMUR5

Calls To: None

References*: Van Vledder (1983a,b)
Kimura (1980)
Goda (9185)

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* References in appendixes are cited in References at the end of the main text.

Subroutine INPUT

Description: Queries user for five input parameters for Kimura's wave group analysis.

Calling Statement: SUBROUTINE INPUT(RHH1,NH,DELH,HM,HC)

Arguments:

RHH1	R*4	Correlation coefficient
NH	I*2	Total number of wave height measurements or total number of intervals of dummy wave height parameters H1 and H2 between zero and upper wave height HU in transition probabilities
DELH	R*4	Delta wave height increment, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
HM	R*4	Mean wave height
HC	R*4	Cutoff or threshold wave height

Called By: Program KIMUR5

Calls To: None

Reference: None

Subroutine KAPPA

Description: Calculates correlation parameter Kappa given correlation coefficient using series approximation method of Battjes for Complete Elliptic Integrals of 1st & 2nd kind.

Calling Statement: SUBROUTINE KAPPA (RHHI,K)

Arguments:

RHH1	R*4	Correlation coefficient
K	R*4	Correlation parameter

Called By: Program KIMUR5

Calls To: None

Reference: Van Vledder (1983a,b)

Subroutine OUTPT

Description: Outputs results from Kimura's algorithms for calculated group run length statistics to disk file, logical unit 2, KIMUR.OUT.

Calling Statement: SUBROUTINE OUTPT(RHH1,NH,DELH,HM,HC,K,Q,P,P11,P22,PHR, J1M,SIGJ1,PTR,J2M,SIGJ2)

Arguments:

RHH1	R*4	Correlation coefficient
NH	I*2	Total number of wave height measurements
		Total number of intervals of dummy wave height parameters H1 and H2 between zero and upper wave height HU in transition probabilities
DELH	R*4	Delta wave height increment between successive H1 and H2 dummy wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
HM	R*4	Mean wave height
HC	R*4	Cutoff or threshold wave height
Q	R*4	Rayleigh one-dimensional (1-D) PDF Q(H1)
P	R*4	Rayleigh two-dimensional (2-D) PDF P(H1,H2)
P11	R*4	Transition probability, neither H1 nor H2 successive wave height exceeds threshold wave height HC
P22	R*4	Transition probability, both H1 and H2 successive wave heights exceed threshold wave height HC
PHR	R*4	Probability of run length having length of J1, P1(J)
J1M	R*4	Mean run length for a run of high waves
SIGJ1	R*4	Standard deviation of run of length J1 for run of high waves
PTR	R*4	Probability of total run length having length of J2, P2(J)
J2M	R*4	Mean total run length
SIGJ2	R*4	Standard deviation of run of length J2 for total run length

Called By: Program KIMUR5

Calls To: None

Reference: None

Subroutine QSF

Description:		Computes vector of integral values for a given equidistant table of function values. Computes integral of function contained in array Y of dimension NDIM for equidistant X-array values spaced H apart using combination of Simpson's and Newton's 3/8 Rules.

Calling Statement: SUBROUTINE QSF(H,Y,Z,NDIM)

Arguments:

H	R*4	Increment of argument values, DELH for x-axis array
Y	R*4	Input vector of function values
Z	R*4	Resulting vector of integral values, contains integrated area under curve represented by array Y
NDIM	I*2	Dimension of vectors Y and Z

Called By: Subroutine RTPI

Calls To: None

References: NRCC Computation Center
Hildebrand (1956)
Zurmehl (1963)

Subroutine RJPDF

Description:		Calculates 2-D, joint, or Bivariate Rayleigh Probability Density Function P(H1,H2) based on Kimura's theory. Uses modified Bessel function of zero order.

Calling Statement: SUBROUTINE RJPDF(NH,DELH,HM,K,P)

Arguments:

NH	I*2	Total number of intervals of dummy wave height parameters H1 and H2 between zero and upper wave height HU in transition probabilities
DELH	R*4	Delta wave height increment between successive H1 and H2 dummy wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
HM	R*4	Mean wave height
K	R*4	Correlation parameter
P	R*4	2-D Rayleigh PDF P(H1,H2)

Called By: Program KIMUR5

Calls To: Subroutine BESI (NRCC Scientific Library Subroutine)

References: Van Vledder (1983a,b)
Kimura (1980)
Goda (1985)

Subroutine RPDF

Description: Calculates 1-D Rayleigh Probability Density Function (PDF) Q(H1) using numerical integration.

Calling Statement: SUBROUTINE RPDF(NH,DELH,HM,Q)

Arguments:

NH	I*2	Total number of intervals of dummy wave height parameter H1 between zero and upper wave height HU in transition probabilities
DELH	R*4	Delta wave height increment between successive H1 wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
HM	R*4	Mean wave height
Q	R*4	1-D Rayleigh PDF

Called By: Program KIMUR5

Calls To: None

References: Van Vledder (1983a,b)
Kimura (1980)
Goda (1985)

Subroutine RTP

Description: Calculates Rayleigh Transition probabilities P11 and P22 given 1-D and 2-D Rayleigh PDF's.

Calling Statement: SUBROUTINE RTP(NH,DELH,HC,Q,P,P11,P22)

Arguments:

NH	I*2	Total number of intervals of dummy wave height parameters H1 and H2 between zero and upper wave height HU in transition probabilities
DELH	R*4	Delta wave height increment between successive H1 and H2 dummy wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
HC	R*4	Cutoff or threshold wave height
Q	R*4	Rayleigh 1-D PDF Q(H1)
P	R*4	Rayleigh 2-D PDF P(H1,H2)
P11	R*4	Transition probability, neither H1 nor H2 successive wave height exceeds threshold wave height HC
P22	R*4	Transition probability, both H1 and H2 successive wave heights exceed threshold wave height HC

Called By: Program KIMUR5

Calls To: Subroutine RTPI

References: Van Vledder (1983a,b)
Kimura (1980)
Goda (1985)

Subroutine RTPI

Description: Integrates 1-D and 2-D Rayleigh PDF Q(H1) and P(H1,H2), respectively.

Calling Statement: SUBROUTINE RTPI(NL,NU,DELH,Q,P,PROB)

Arguments:

NL	I*2	Lower integration limit array element
NU	I*2	Upper integration limit array element
DELH	R*4	Delta wave height increment between successive H1 or H2 dummy wave heights, controls upper wave height integration limit, HU = NH * DELH in transition probabilities
P	R*4	2-D Rayleigh PDF P(H1,H2)
Q	R*4	1-D Rayleigh PDF Q(H1)
PROB	R*4	Transition Probability, either P11 or P22

Called By: Subroutine RTP

Calls To: Subroutine QSF (NRCC Scientific Library Subroutine)

References: Van Vledder (1983a,b)
Kimura (1980)
Goda (1985)

Subroutine TRUN

Description: Calculates total run group statistics of probability for different run lengths, mean run length, and standard deviation of run length.

Calling Statement: SUBROUTINE TRUN(P11,P22,PTR,J2M,SIGJ2)

Arguments:

P11	R*4	Transition probability, neither H1 nor H2 successive wave heights exceed threshold wave height HC
P22	R*4	Transition probability, both H1 and H2 successive wave heights exceed threshold wave height HC
PTR	R*4	Probability of total run length having length of J2, P2(J)
J2M	R*4	Mean total run length
SIGJ2	R*4	Standard deviation of run of length J2 for total run length

Called By: Program KIMUR5

Calls To: None

References: Van Vledder (1983a,b)
Kimura (1980)
Goda (1985)

APPENDIX D: NOTATION

$E[\ ]$	Expectation operator
$E(\ )$	Complete elliptic integral of the second kind
f	Frequency variable
H	Wave height variable
H_i	Dummy wave height variable
H_{i+1}	Successive wave height variable
H_1	First of two successive wave heights dummy variable
H_2	Second of two successive wave heights dummy variable
H_c	Cutoff or threshold wave height
H_m	Mean wave height
H_{max}	Maximum wave height
H_{med}	Median wave height
H_r	RMS wave height
H_s	Significant wave height
H_*	Cutoff or threshold wave height
$I_o[\ ]$	Modified Bessel function of zeroth order
j_1	Run length for run of high waves, i.e. 1,2,3,...11,12+
j_2	Run length for total run, i.e. 2,3,4,...11,12+
$\overline{j_1}$	Mean run length for run of high waves
$\overline{j_2}$	Mean run length for total run
k	Lag of autocorrelation function estimate
$K(\ )$	Complete elliptic integral of the first kind
m_o	Zeroth moment of time series of wave elevations
N	Total number of points in wave height time series
p	Probability that wave height H exceeds threshold wave height or Markov Chain transition probability matrix
P_{11}	Transition probabilities--neither H_1 nor H_2 exceeds threshold height
P_{22}	Transition probabilities--both H_1 and H_2 exceed threshold height; simultaneous exceedance of threshold wave height by both H_1 and H_2 waves
$P(H_1,H_2)$	Joint or bivariate Rayleigh probability density function
P_n	Markov Chain distribution after n-time transitions
P_0	Initial Markov Chain distribution
$P(j_1)$	Run length probability for run of high waves
$P(j_2)$	Run length probability for total run

q	Probability that wave height H does not exceed threshold wave height
Q_p	Goda's spectral peakedness factor
$q(H_1)$	Rayleigh probability density function for individual wave heights
$R_{hh}(1)$	Correlation coefficient for successive wave heights
$S(f)$	Spectral estimate of the surface elevation
T_m	Mean zero-crossing wave period
κ	Correlation parameter, equals 2ρ
ρ	Correlation parameter, equals $\kappa/2$
π	3.14159. . .
γ_h	Correlation coefficient for successive wave heights
σ_H	Standard deviation of wave height time series
$\sigma(j_1)$	Standard deviation for run of high waves
$\sigma(j_2)$	Standard deviation for total run

END
Feb.
1988
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